Lesson 5.6 Cont.---The Discriminant

The discriminant is part of the Quadratic Formula that you can use to determine the number of real roots of a quadratic equation.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

<table>
<thead>
<tr>
<th>( b^2 - 4ac &gt; 0 )</th>
<th>( b^2 - 4ac = 0 )</th>
<th>( b^2 - 4ac &lt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>two distinct real solutions</td>
<td>one distinct real solution</td>
<td>two distinct nonreal complex solutions</td>
</tr>
</tbody>
</table>

**Caution!**

Make sure the equation is in standard form before you evaluate the discriminant, \( b^2 - 4ac \).

**Example:**

Find the type and number of solutions for the equation.

1. \( x^2 + 36 = 12x \)
   - \( x^2 - 12x + 36 = 0 \)
   - \( a = 1 \)
   - \( b = -12 \)
   - \( c = 36 \)
   - \( b^2 - 4ac = (-12)^2 - 4(1)(36) = 144 - 144 = 0 \)
   - one real solution

2. \( x^2 + 40 = 12x \)
   - \( x^2 - 12x + 40 = 0 \)
   - \( a = 1 \)
   - \( b = -12 \)
   - \( c = 40 \)
   - \( b^2 - 4ac = (-12)^2 - 4(1)(40) = 144 - 160 = -16 \)
   - two different complex solutions

3. \( x^2 + 30 = 12x \)
   - \( x^2 - 12x + 30 = 0 \)
   - \( a = 1 \)
   - \( b = -12 \)
   - \( c = 30 \)
   - \( b^2 - 4ac = (-12)^2 - 4(1)(30) = 144 - 120 = 24 \)
   - two different real solutions
The graph shows related functions. Notice that the number of real solutions for the equation can be changed by changing the value of the constant $c$.

Example:
An athlete on a track team throws a shot put. The height $y$ of the shot put in feet $t$ seconds after it is thrown is modeled by $y = -16t^2 + 24.6t + 6.5$. The horizontal distance $x$ in between the athlete and the shot put is modeled by $x = 29.3t$. To the nearest foot, how far does the shot put land from the athlete?

$t = 1.77 \text{ sec on calculator}$

$x = 29.3t$
$x = 29.3(1.77)$
$x = 51.86 \approx 52 \text{ ft}$
# Properties of Solving Quadratic Equations

<table>
<thead>
<tr>
<th>Method</th>
<th>When to Use</th>
<th>Examples</th>
</tr>
</thead>
</table>
| Graphing             | Only approximate solutions or the number of real solutions is needed. | \(2x^2 + 5x - 14 = 0\)  
\[x \approx -4.2\text{ or } x \approx 1.7\] |
| Factoring            | \(c = 0\) or the expression is easily factorable. | \(x^2 + 4x + 3 = 0\)  
\[(x + 3)(x + 1) = 0\]  
\[x = -3\text{ or } x = -1\] |
| Square roots         | The variable side of the equation is a perfect square. | \( (x - 5)^2 = 24\)  
\[\sqrt{(x - 5)^2} = \pm \sqrt{24}\]  
\[x - 5 = \pm 2\sqrt{6}\]  
\[x = 5 \pm 2\sqrt{6}\] |
| Completing the square| \(a = 1\) and \(b\) is an even number. | \(x^2 + 6x = 10\)  
\[x^2 + 6x + \boxed{9} = 10 + \boxed{9}\]  
\[x^2 + 6x + \left(\frac{6}{2}\right)^2 = 10 + \left(\frac{6}{2}\right)^2\]  
\[(x + 3)^2 = 19\]  
\[x = -3 \pm \sqrt{19}\] |
| Quadratic Formula    | Numbers are large or complicated, and the expression does not factor easily. | \(5x^2 - 7x - 8 = 0\)  
\[x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-8)}}{2(5)}\]  
\[x = \frac{7 \pm \sqrt{209}}{10}\] |

## Helpful Hint

No matter which method you use to solve a quadratic equation, you should get the same answer.
Lesson Quiz:
Find the type and member of solutions for each equation.

1. $x^2 - 14x + 50$
2. $x^2 - 14x + 48$

3. A pebble is tossed from the top of a cliff. The pebble’s height is given by $y(t) = -16t^2 + 200$, where $t$ is the time in seconds. Its horizontal distance in feet from the base of the cliff is given by $d(t) = 5t$. How far will the pebble be from the base of the cliff when it hits the ground?

\[ t = 3.54 \text{s} \]
\[ d = 5(3.54) = 17.7 \text{ ft} \]

HW: p. 361 30-35, 37a, 49-51, 53, 61-64, 78 = 16 problems